

Herd formation and information transmission in a population: non-universal behaviour

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Abstract. We present generalized dynamical models describing the sharing of information, and the corresponding herd behavior, in a population based on the recent model proposed by Eguíluz and Zimmermann (EZ) [Phys. Rev. Lett. **85**, 5659 (2000)]. The EZ model, which is a dynamical version of the herd formation model of Cont and Bouchaud (CB), gives a reasonable model for the formation of clusters of agents and for actions taken by clusters of agents. Both the EZ and CB models give a cluster size distribution characterized by a power law with an exponent $-5/2$. By introducing a size-dependent probability for dissociation of a cluster of agents, we show that the exponent characterizing the cluster size distribution becomes model-dependent and non-universal, with an exponential cutoff for large cluster sizes. The actions taken by the clusters of agents generate the price returns, the distribution of which is also characterized by a model-dependent exponent. When a size-dependent transaction rate is introduced instead of a size-dependent dissociation rate, it is found that the distribution of price returns is characterized by a model-dependent exponent while the exponent for the cluster-size distribution remains unchanged. The resulting systems provide simplified models of a financial market and yield power law behaviour with an easily tunable exponent.

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1 Introduction

Herd behaviour and information transmission among agents are obviously important in financial markets. With the newly established area of econophysics [1,2], various multi-agent models of financial markets have been studied [3,4]. In agent-based models, markets are typically modelled by a population of competing agents. Through information transmission and opinion sharing, these agents may not act independently. The collective behaviour of clusters of agents, also referred to as crowds, in which there is efficient information sharing among the agents is an important factor in both real and simulated markets [5]. These crowds are dynamic in nature in that there is a continual process of crowd formation and dissociation.

Recently, Eguíluz and Zimmermann [6] proposed and studied a simple model for stochastic opinion cluster formation and information dispersal. It is a dynamical model

(henceforth referred to as the EZ model) in which there is a continual grouping and re-grouping of agents to form clusters. A cluster or crowd of agents act together (either buying or selling) and then dissolve after the transaction has occurred. When a cluster of agents decides not to trade, *i.e.*, an inactive state, it may combine with another cluster of agents to form a bigger cluster. Detailed numerical studies [6] and mean field analysis [7] revealed several interesting features of the model. For example, it was observed that the model could lead to a fat-tail distribution of price returns qualitatively similar to that observed in real markets [1,8]. In addition, the cluster size distribution n_s shows a scaling behaviour of the form $n_s \sim s^{-5/2}$ for a range of cluster size s , followed by an exponential cutoff [7]. The EZ model represents a dynamical version of the static percolation-type model of Cont and Bouchaud [9] in which herd formation is described by random connection between agents, and the cluster size distribution is found to follow the same scaling behaviour and with the same value of $-5/2$ for the exponent. Several variations on the model have been proposed and studied.

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These variations include the spreading of opinion to multiple clusters [7] and inhomogeneous dissociation of clusters [10]. Interestingly, it was found that the values of the exponent characterizing the cluster size distribution seem to be robust against these variations, *i.e.*, these values remain unchanged for the different variations proposed so far. We note that similar scaling behavior has been found in the size distribution of businesses [11,12], although the value of the exponent is different.

Here we introduce two generalized versions of the EZ model. One version has a size-dependent probability for the dissociation of clusters after opinion sharing. The distributions of cluster sizes and price returns are found to be characterized by model-dependent exponents. Another version has a size-dependent transaction rate. It leads to a price returns distribution with a model-dependent exponent, while keeping the exponent for the cluster-size distribution to be $-5/2$. Both models can be treated analytically *via* a mean field approach. The mean field results are found to be in good agreement with numerical results. The EZ model is re-covered as a special case of our generalized models.

The plan of the paper is as follows. We define our generalized model with size-dependent cluster dissociation probability in Section 2 and present the numerical results of the model. Section 3 provides a mean field analysis. The analytic results are compared with numerical simulations. In Section 4, we define another generalized model with a size-dependent transaction rate and present numerical results. The results are summarized in Section 5.

2 Model I: size dependent cluster dissociation rate

We consider a model with a total of N agents. Following reference [6], a cluster or crowd is a group of agents who can exchange information efficiently. These agents make the same decision at a given moment in time. Initially, all the agents are isolated. As time evolves, an agent belongs to a cluster of a certain size. At each timestep an agent, say the i th one, is chosen at random. Let s_i be the size of the cluster to which the agent belongs. Since the agents within a cluster have a common opinion, all agents in such a cluster tend to imitate each other and hence act together. With probability a the agent, and hence the whole cluster, decides to make a transaction, *e.g.* to buy or to sell with equal probability. After the transaction, the cluster is then broken up into isolated agents with a probability $s_i^{-\delta}$, with $0 \leq \delta < 1$. With probability $(1 - a)$ the agents decide not to make a transaction, *i.e.*, they wait and try to gather more information. The other agents in the cluster follow. In this case, another agent j is chosen at random. The two clusters of sizes s_i and s_j then either combine to form a bigger cluster with probability $s_i^{-\delta} s_j^{-\delta}$, or the two clusters remain separate with probability $(1 - s_i^{-\delta} s_j^{-\delta})$. Here a can be treated as a parameter reflecting the investment rate showing how frequent a transaction is made. Our model thus represents a generalization of the basic EZ model to

the case in which a cluster of agents may stay together to form a group *after* making a transaction. The probability of dissociation $s_i^{-\delta}$ implies that larger clusters have a larger tendency to remain grouped while smaller clusters are easier to break up [13]. For the special case of $\delta = 0$, our model reduces to the EZ model.

In the EZ model ($\delta = 0$), clusters of agents break up after a transaction. Here, our model includes a dissociation probability depending on the cluster size – this feature may be invoked to mimic practical aspects of a financial market, such as the effect of news arrival. Imagine one of the agents in a cluster of size s_i receives some external news with probability a at a given timestep. This external news suggests that he, and hence the other members of his cluster, should immediately trade (buy or sell). Since the news is external, the crowd act together in this one moment, leaving the cluster with a finite probability of subsequently dissociating. Suppose that they sense, *e.g.* from the resulting price-movement, that they are a member of a large crowd of like-minded agents: in practice many traders like to feel part of a larger crowd for reassurance. We therefore assume that the crowd breaks up with a size-dependent probability $p(s_i)$, where $p(s_i) = 1$ for $s_i = 1$ and $p(s_i)$ decreases monotonically as s_i increases. By contrast, with probability $(1 - a)$ there is no news arrival from outside. The agent in the chosen cluster, uncertain about whether to buy or sell, makes contact with an agent in another cluster of size s_j . The agents share information and come up with a new opinion. Each of them then separately tries to persuade the other members of his cluster of the new opinion. With probability $p(s_i)$ ($p(s_j)$) the opinion of cluster i (j) changes to the new opinion. Thus, the two clusters combine with probability $p(s_i)p(s_j)$. It turns out that this particular form of the two combined modifications to the EZ model, can be treated analytically using our mean field analysis. As a specific example, our numerical simulations are carried out for the case in which $p(s) \sim s^{-\delta}$.

Let n_s be the number of clusters of size s . Figure 1 shows the results of numerical simulations on the cluster size distribution in the steady state, for various values of the parameter δ . The results are obtained for a system with $N = 10^4$ agents and $a = 0.3$ [6]. Averages are taken over a time window of 10^6 time steps after the transient behavior has disappeared, together with a configuration average over 100 different runs with different initial conditions. The $\delta = 0$ results give the features in the EZ model. For a range of s , $n_s \sim s^{-\beta}$ with $\beta = 5/2$ [6]. Deviation from the scaling behavior sets in at a value of s depending on the value of the parameter a . For smaller values of a , the scaling region enlarges. These features are consistent with previous numerical [6] and analytical studies [7]. For $0 \leq \delta < 1$, it is observed that the size distribution n_s still scales with s in a range of s as in the EZ model. However, the exponent becomes model dependent and hence *non-universal*. The data shows that the exponent δ is consistent with the behavior $n_s \sim s^{-\beta(\delta)}$, where $\beta(\delta) = 5/2 - \delta$. A mean field analysis can be used to extract this scaling behavior, as will be described in the next section.

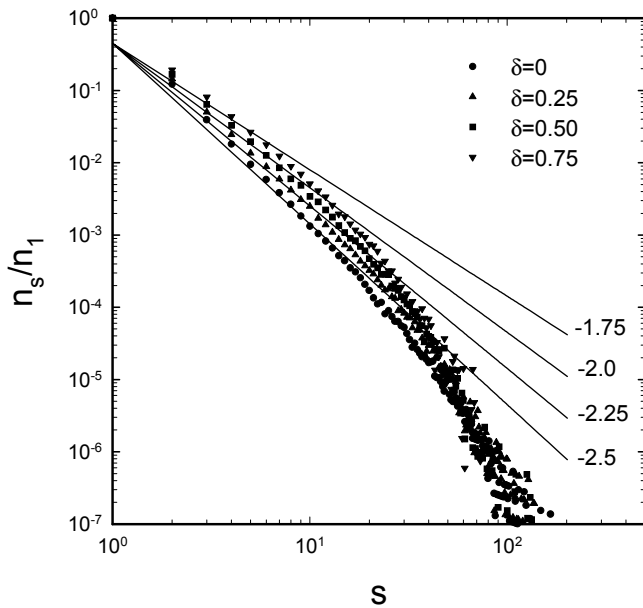


Fig. 1. The size distribution n_s/n_1 as a function of the size s on a log-log scale for different values of δ obtained by numerical simulations (symbols) of Model I. The values of δ used in the calculations are: $\delta = 0, 0.25, 0.50, 0.75$. The solid lines are a guide to the eye corresponding to exponents $\beta = 2.5, 2.25, 2.0, 1.75$ respectively.

It is interesting to note that several attempts have been made to modify the EZ model. These extensions include, for example, the study by d’Hulst and Rodgers on democracy *versus* dictatorship by incorporating an inhomogeneous investment rate in the population [10] and also allowing rumor to spread to multiple clusters in one time step after a chosen cluster decides not to make a transaction [7]. All the extensions proposed so far give $\beta = 5/2$, hence the value seems to be robust. The present model incorporates a size-dependent dissociation probability of a cluster after a transaction and leads to a tunable and model-dependent $\beta(\delta)$. Thus our model actually gives a set of models with different values of β , similar to the case of changing a system from one universality class to another in problems in critical phenomena. In fact, the situation is reminiscent of the non-universal exponent of conductivity in continuum percolation [14, 15]. In percolation problems [16], it is known that the effective conductivity for a system consisting of insulators and conductors exhibits the scaling behavior $\sigma_e \sim (p - p_c)^t$ near the percolation threshold p_c . The exponent t is universal in that its value depends only on the dimension of the system, regardless of other details, *e.g.* lattice type. However, if the conductances σ_e of the conductors follow a distribution of the form $P(\sigma_e) \sim \sigma_e^{-\delta}$ with $0 < \delta < 1$, the t -exponent [14] and other related properties [15, 17] become non-universal with exponents taking on a value depending on δ . It should be noted that it is not so surprising to see a connection between percolation and model for herd behavior. In the model of Cont and Bouchaud [9], the EZ model [6] and their variations [7, 10], an agent could be connected to any

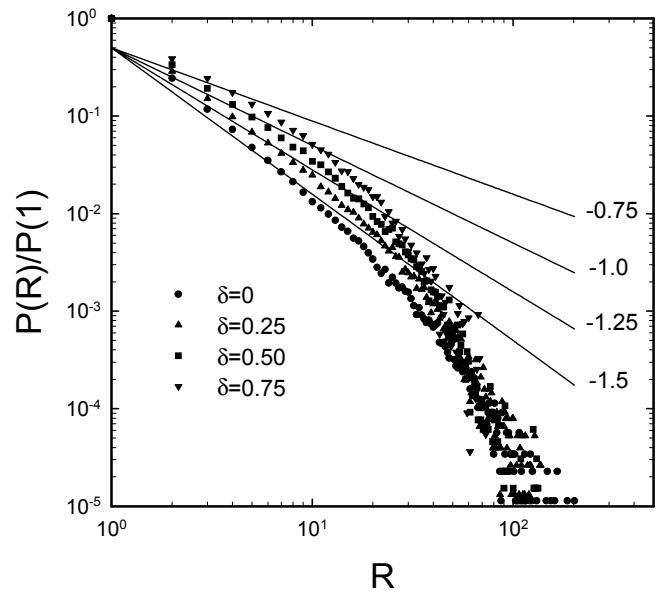


Fig. 2. The distribution of price returns $P(R)/P(1)$ as a function of R on a log-log scale for different values of δ (symbols) in Model I. The values of δ used in the calculations are: $\delta = 0, 0.25, 0.50, 0.75$. The solid lines are a guide to the eye corresponding to exponents $\alpha = 1.5, 1.25, 1.0, 0.75$ respectively.

one of $(N-1)$ other agents to form a cluster. These models hence represent a problem of connectivity in high dimensions. Several other percolation type models [18–20] have also been proposed to explain the phenomena observed in real markets. It is also interesting to note that value of power law exponents in statistical physics could also be made non-universal by introducing long-range interactions. By making the larger clusters harder to dissociate, it can be thought of as effectively introducing some long range interaction among agents in time.

Eguíluz and Zimmermann [6] applied their model to study the distribution of price returns. A price can be generated according to

$$P(t+1) = P(t) \exp(s'/\lambda), \quad (1)$$

where λ is a parameter for the liquidity of the market. The price return $R(t) = \ln P(t) - \ln P(t-1)$ is defined to be the relative number of agents buying or selling at a time with $s' = s$ for a cluster of agents deciding to buy, and $s' = -s$ for a cluster deciding to sell at a given timestep. Numerical results for the EZ model showed that the distribution of returns $P(R) \sim R^{-\alpha}$ with $\alpha = 3/2$. We have carried out similar calculations for our model. Figure 2 shows the price return distributions for different values of δ on a log-log scales. As for the cluster size distribution, the exponent α is now *non-universal* and takes on the value $3/2 - \delta$, which is also the value of $\beta(\delta) - 1$ [6].

$$n_s = \frac{(2s-2)!(1-a)^{s-1} \left(\sum_{r=1}^{\infty} r^{1-\delta} n_r \right)^s [(1-a) \sum_{r=1}^{\infty} r^{1-\delta} n_r + Na]^s}{(s!)^2 s^{-\delta} [Na + 2(1-a) \sum_{r=1}^{\infty} r^{1-\delta} n_r]^{2s-1}}. \quad (12)$$

3 Mean field analysis

The cluster size distribution in the EZ model can be studied *via* a mean field analysis [7]. The treatment can be extended to the generalized model to extract the scaling behavior of n_s , though the algebra is more complicated. Let $n_s(t)$ be the number of clusters of size s at time t . The dynamics of $n_s(t)$ is governed by the following master equation describing the result of collection action of the members of the cluster. The equation is

$$N \frac{\partial n_s}{\partial t} = -a s^{1-\delta} n_s + \frac{(1-a)}{N} \sum_{r=1}^{s-1} r^{1-\delta} n_r (s-r)^{1-\delta} n_{s-r} - \frac{2(1-a) s^{1-\delta} n_s}{N} \sum_{r=1}^{\infty} r^{1-\delta} n_r \quad (2)$$

for $s > 1$. Each of the terms on the right hand side of equation (2) represents the consequence of a possible action of the agent. The first term describes the dissociation of a cluster of size s after a transaction is made. The second term represents coagulation of two clusters to form a cluster of size s . The third term represents the combination of a cluster of size s with another cluster. For clusters of size unity ($s = 1$), we have

$$N \frac{\partial n_1}{\partial t} = a \sum_{r=2}^{\infty} r^{2-\delta} n_r - \frac{2(1-a)n_1}{N} \sum_{r=1}^{\infty} r^{1-\delta} n_r. \quad (3)$$

Here, the first term comes from the dissociation of any clusters into isolated agents and the second term describes the combination of a cluster of size unity with another cluster. In the steady state, $\frac{\partial n_s}{\partial t} = 0$, we have

$$s^{1-\delta} n_s = A \sum_{r=1}^{s-1} r^{1-\delta} (s-r)^{1-\delta} n_r n_{s-r} \quad (4)$$

for $s > 1$, and

$$n_1 = B \sum_{r=2}^{\infty} r^{2-\delta} n_r \quad (5)$$

for $s = 1$, where

$$A = \frac{1-a}{Na + 2(1-a) \sum_{r=1}^{\infty} r^{1-\delta} n_r}, \quad (6)$$

and

$$B = \frac{Na}{2(1-a) \sum_{r=1}^{\infty} r^{1-\delta} n_r}. \quad (7)$$

The aim here is to extract the scaling behaviour. Invoking a generating function approach [21], we let

$$G(\omega) = \sum_{r=0}^{\infty} r^{1-\delta} n_r e^{-\omega r} = g(\omega) + n_1 e^{-\omega}, \quad (8)$$

where $g(\omega) = \sum_{r=2}^{\infty} r^{1-\delta} n_r e^{-\omega r}$. It follows from equation (4) that the function $g(\omega)$ satisfies the equation

$$g^2(\omega) + \left(2n_1 e^{-\omega} - \frac{1}{A} \right) g(\omega) + n_1^2 e^{-2\omega} = 0. \quad (9)$$

Note that A can be expressed in terms of n_1 and $g(0)$, and

$$g(\omega) = \frac{1}{4A} \left(1 - \sqrt{1 - 4n_1 A e^{-\omega}} \right)^2. \quad (10)$$

The number of clusters of size s can be found formally by

$$n_s = \frac{1}{s^{1-\delta} s!} \left. \frac{\partial^s G}{\partial z^s} \right|_{z=0}, \quad (11)$$

where $z = e^{-\omega}$. The resulting expression for n_s is

See equation (12) above.

Invoking Sterling's formula yields

$$n_s \sim N \left[\frac{4(1-a) \left[(1-a) + \frac{Na}{\sum_{r=1}^{\infty} r^{1-\delta} n_r} \right]}{\left[\frac{Na}{\sum_{r=1}^{\infty} r^{1-\delta} n_r} + 2(1-a) \right]^2} \right]^s s^{-(\frac{5}{2}-\delta)}. \quad (13)$$

For $\delta = 0$, the sum $\sum_{r=1}^{\infty} r^{1-\delta} n_r = N$ and the previous results of references [6,7] are recovered. For $\delta \neq 0$, it is difficult to solve for n_s . Since the summations in equations (12, 13) give a number, our result shows that $n_s \sim s^{-\beta(\delta)}$ with $\beta(\delta) = 5/2 - \delta$ for the present model. This δ -dependent exponent is also indicated in Figure 1 (lines are a guide to the eye). We note that the scaling behaviour is masked by the behaviour of the term in the squared brackets in equation (13) for large values of s , similar to the situation for the EZ model [7].

4 Model II: size-dependent transaction rate

In this generalization of the EZ model, we introduce a size-dependent transaction rate instead of size-dependent dissociation and combination rates of clusters. It is motivated by the fact that it is hard to measure the structural properties of herds, *e.g.* the size distribution. Instead, the resulting properties of the herd effect, *e.g.* price returns, are easier to obtain [1,8]. Empirically, it was found that the distribution of price returns, while taking on a power

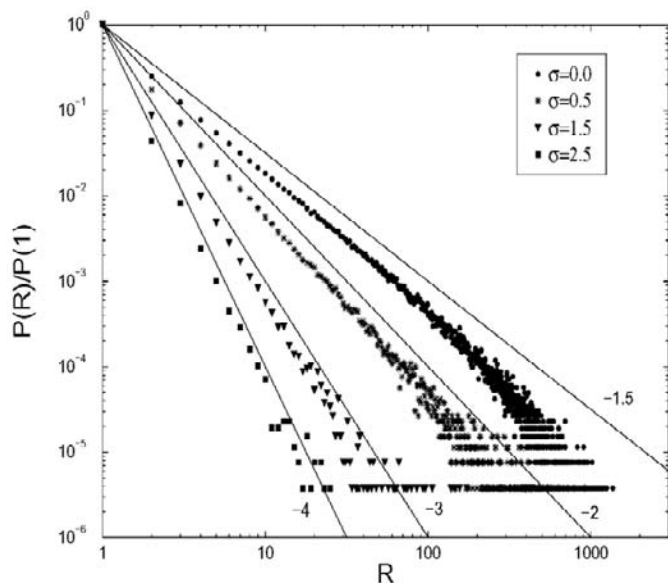


Fig. 3. The distribution of price returns as a function of σ (symbols) in Model II. The values of σ used in the calculations are: $\sigma = 0, 0.5, 1.5, 2.5$. The solid lines are a guide to the eye corresponding to exponents $\alpha = 1.5, 2, 3, 4$ respectively.

law behaviour, has an exponent α that is deviated from the value of $3/2$ in the EZ model. It is therefore interesting to investigate models in which the exponent for the distribution of the price returns becomes tunable. Here, we introduce a model in which the EZ dynamics is used for cluster formation and dissociation, *i.e.*, the cluster always dissolves after an action is taken. However, when an agent i is randomly chosen, there are three possible actions. With probability $a(1 - s_i^{-\sigma})$, the cluster of agents decide not to take an action (make a transaction) and the cluster simply dissolves. With probability $as^{-\sigma}$, the cluster of agents act collectively to either buy or sell, and the cluster dissolves after the transaction is made. With probability $(1 - a)$, the chosen cluster combines with another randomly picked cluster. For $\sigma = 0$, the present model reduces to the EZ model. For $\sigma \neq 0$, the model reflects that while agents may have established connections to exchange information through different means, they may tend to look for common opinion before taking actions. Reaching a consensus becomes difficult as the cluster size increases. Therefore the effective transaction rate is lower than a and becomes size-dependent.

Since the combination and dissociation probabilities are identical to those in the EZ model, the distribution of cluster sizes also follows $n_s \sim s^{-5/2}$, with an exponential cutoff as in the EZ model. Figure 3 shows the distribution of price returns in Model II for different values of σ in a system with $N = 10^4$ agents and $a = 0.1$. It is noted that the return $P(R)$ follows a power law $P(R) \sim R^{-\alpha}$, where the exponent α has become model-dependent. The value of α is well described by $\alpha = 3/2 + \sigma$. The result can be understood as the distribution of returns is given by the product of the probability of selecting a cluster of size

s times the probability of making a transaction [6], *i.e.*, $P(R) \sim R^{-\alpha} \sim s^{-5/2}s^{-\sigma} \sim s^{-(3/2+\sigma)}$. Since the price returns are generated by the collective action of the whole cluster, we have $P(R) \sim R^{-(3/2+\sigma)}$.

5 Summary

In summary, we have proposed and studied the cluster size distribution, and the price returns distribution, of two generalized versions of the EZ model for herd behavior and information sharing in a population. We have carried out numerical simulations on our models. The two generalizations can be treated analytically within a mean field approach. By introducing a probability for dissociation of a cluster depending on its size, the exponent characterizing the cluster size distribution takes on a model-dependent non-universal value, with a corresponding shift in the exponent characterizing the price returns from the value of the EZ model. It is also possible to make the exponent for the distribution of price returns model-dependent while keeping the exponent of the cluster-size distribution unchanged by imposing a size-dependent transaction rate. Our models thus illustrate that it is possible to tune the power law behaviour. It is interesting to note that observed values for the exponent α in markets can be as large as 4 [8], which is quite different from the robust value of $3/2$ in the EZ model. Within our model, a value of α with $\alpha > 3/2$ can be attributed to the hesitation among agents in a cluster in making a transaction.

Several extensions of our models are immediately possible. Our particular choices of the form of the probability for dissociation of clusters in Model I and the probability for making a transaction in Model II allow us to tune the exponent. Exploring other functional forms of the probabilities may alter the range of values in which the exponent can be tuned. Our model can also be extended to study the size distribution of businesses. A model similar to the EZ model has already been proposed in this context [13]. However the scaling behavior seems to be non-universal for data from different countries [11]. Our work thus provides a possible extension to cope with this observed non-universality.

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